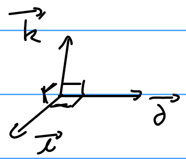
Mathématiques pour la cinématique

Espace euclidien à 3D.

1. Base orthonormée.



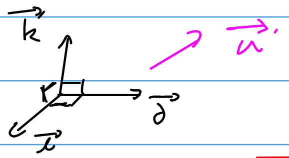
$(\vec{i}, \vec{j}, \vec{k})$: base orthonormée directe

normée : $\|\vec{i}\| = \|\vec{j}\| = \|\vec{k}\| = 1$

orthogonal : $\vec{i}.\vec{j} = 0$, $\vec{i}.\vec{k}$, $\vec{j}.\vec{k} = 0$

direct : $\vec{k} = \vec{i} \wedge \vec{j}$

2. Composante d'un vecteur dans une base

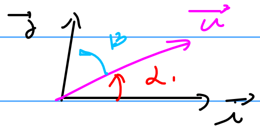


$$\vec{u} = a\vec{i} + b\vec{j} + c\vec{k} \iff \vec{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}_{(\vec{i},\vec{j},\vec{k})}$$

$$\boxed{\begin{aligned} a &= \vec{u}.\vec{i} \\ b &= \vec{u}.\vec{j} \\ c &= \vec{u}.\vec{k} \end{aligned}}$$

composants de $\vec{u}$ ds $(\vec{i}, \vec{j}, \vec{k})$

Application : espace à 2D, $\|\vec{u}\| = u$.



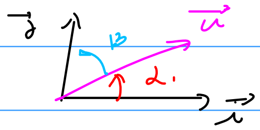
Composants de $\vec{u}$ ds $(\vec{i}, \vec{j})$?

$$\vec{u} = (\vec{u}.\vec{i})\,\vec{i} + (\vec{u}.\vec{j})\,\vec{j}$$

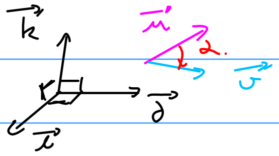
$\vec{u}.\vec{i} = \|\vec{u}\|\,\|\vec{i}\| \cos(\vec{u}, \vec{i}) = u \times 1 \cos\alpha = u \cos\alpha$.

$\vec{u}.\vec{j} = \|\vec{u}\|\,\|\vec{j}\| \cos(\vec{u}, \vec{j}) = u \times 1 \times \cos \underbrace{\beta}_{\frac{\pi}{2} - \alpha} = u \cos\left(\frac{\pi}{2} - \alpha\right) = u \sin\alpha$.

$$\boxed{\vec{u} = u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}}$$



3. Produit scalaire.



Produit scalaire entre $\vec{u}$ et $\vec{v}$.

Expression 1 :

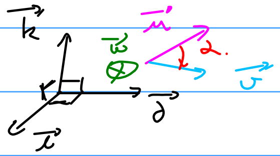
$$\boxed{\vec{u}\cdot\vec{v} = \|\vec{u}\|.\|\vec{v}\|\cos\alpha \quad , \quad \alpha = (\vec{u},\vec{v})}$$

Rem : $\vec{u} \perp \vec{v} \iff \vec{u}\cdot\vec{v} = 0$

Expression 2. : dans la base $(\vec{i},\vec{j},\vec{k})$. $\vec{u}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$ et $\vec{v}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$

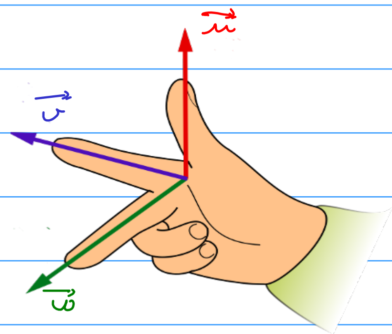
$$\boxed{\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3.}$$

4. Produit vectoriel.



Produit vectoriel : $\vec{w} = \vec{u}\wedge\vec{v}$

Expression 1 : $\vec{w}$ $\begin{cases} w = \|\vec{w}\| = \|\vec{u}\|.\|\vec{v}\|.|\sin\alpha| , \quad \alpha = (\vec{u},\vec{v}) \\ \vec{w}\perp\vec{u} \quad \text{et} \quad \vec{w}\perp\vec{v} \\ \text{sens ?} \quad \underset{m}{\vec{w}} = \underset{p}{\vec{u}} \wedge \underset{i}{\vec{v}} \end{cases}$



Rem : $\vec{v}\wedge\vec{u} = -\vec{u}\wedge\vec{v} = -\vec{w}$

Expression 2. : ds la base $(\vec{i},\vec{j},\vec{k})$

$\vec{u} = \begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$ et $\vec{v}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$

$$\vec{u}\wedge\vec{v} = \begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix} = \begin{pmatrix}u_2v_3 - u_3v_2,\\ -(u_1v_3 - u_3v_1.)\\ u_1v_2 - u_2v_1\end{pmatrix}_{(\vec{i},\vec{j},\vec{k})}$$