S13
Circuits linéaires en régime sinusoïdal
Impédance

Cadre : signaux harmoniques : $s(t) = S_m \cos(\omega t + \varphi)$
$\equiv$ régime sinusoïdal.

1. Impédance complexe

1.1. Définition

$\mathcal{D}$: dipôle linéaire i.e sa loi de fonctionnement est une équat° diff. linéaire à coeff cst.

Conv. récepteur.

Régime sinusoïdal :

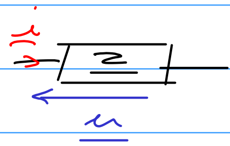
$u(t) = u_m \cos(\omega t + \varphi) \Leftrightarrow \underline{u}(t) = \underline{u_m}\, e^{j\omega t}$

$i(t) = i_m \cos(\omega t) \longleftrightarrow \underline{i}(t) = \underline{i_m}\, e^{j\omega t}$

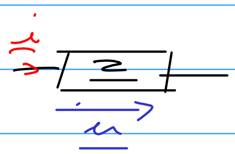
$$\boxed{\underline{u} = \underline{Z}\,\underline{i} \quad \text{et} \quad \underline{u_m} = \underline{Z}\,\underline{i_m}}$$

impédance complexe de $\mathcal{D}$.

Représentat° symbolique :



Rem :



$\underline{u} = -\underline{Z}\,\underline{i}$

1.2. Impédance.

$\underline{Z} = Z e^{j\varphi}$ $\qquad Z = |\underline{Z}|$ impédance.

$\varphi = \arg(\underline{Z})$

1.3. Lien entre u et i

$i(t) = \underline{i_m} \cos \omega t \qquad u(t) = \underline{u_m} \cos(\omega t + \underline{\varphi})$

$\underline{i} = \underline{i_m} e^{j\omega t}, \quad \underline{i_m} = i_m$

$\underline{u} = \underline{u_m} e^{j\omega t}, \quad \underline{u_m} = u_m e^{j\varphi}$

$\underline{Z}$ relie $\underline{u}$ à $\underline{i}$: $\underline{u_m} = \underline{Z}\, \underline{i_m}$

$$\Leftrightarrow \begin{cases} \boxed{u_m = Z\, i_m} \quad (\text{V}, \Omega, \text{A}) \\ \arg \underbrace{\underline{u_m}}_{\varphi} = \arg \underbrace{\underline{i_m}}_{0} + \arg \underbrace{\underline{Z}}_{\varphi} \end{cases}$$

$$\boxed{\varphi = \arg \underline{Z}}$$

déphasage de la tension par rapport au courant.

1.4. Admittance

$$\boxed{\underline{Y} = \frac{1}{\underline{Z}}} \Rightarrow \underline{i} = \underline{Y}\,\underline{u} \quad \text{et} \quad \underline{i_m} = \underline{Y}\,\underline{u_m}$$

admittance complexe.

$Y = |\underline{Y}| \qquad \arg \underline{Y} = -\arg \underline{Z}$

admittance.

1.5. Résistance et réactance.

$\underline{Z} = R + jX$

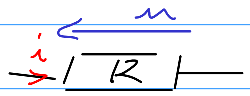
résistance. réactance.

Si $R \neq 0$: le dipôle dissipe de l'énergie (dipôle résistif)

Si $R = 0$: le dipôle ne dissipe pas d'énergie (dipôle réactif).

2. Conducteur ohmique idéal.

2.1. Impédance.



$u = Ri \overset{R.S.}{\Longrightarrow} \underline{u} = R\underline{i}$ et $\underline{u} = \underline{Z}\,\underline{i}$

Donc $\boxed{\underline{Z} = R.}$

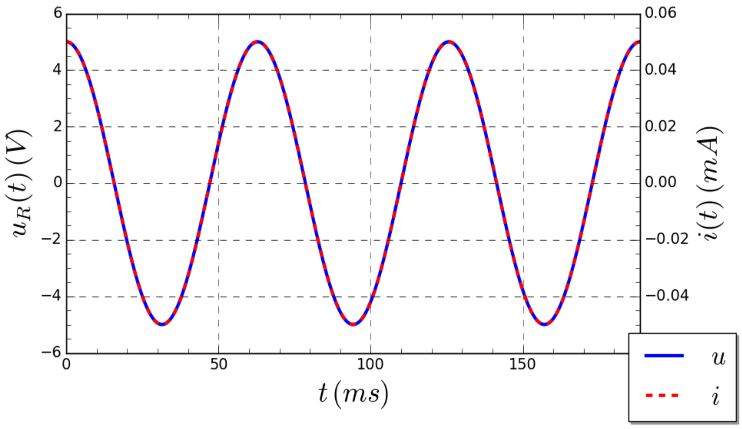
2.2. Lien entre tension et courant

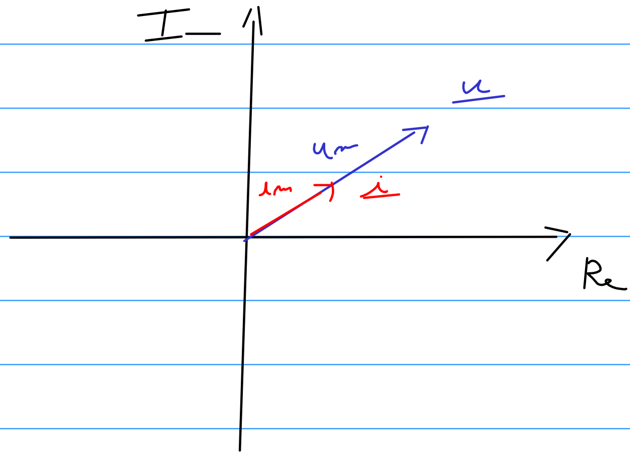
$i(t) = i_m \cos(\omega t)$. $u(t)$? $u(t) = u_m \cos(\omega t + \varphi)$

u_m ? φ ?

$\underline{u_m} = R\,\underline{i_m} \Leftrightarrow \begin{cases} u_m = R i_m \\ \underbrace{\arg(\underline{u_m})}_{\varphi} = \underbrace{\arg(R)}_{0} + \underbrace{\arg \underline{i_m}}_{0} \end{cases}$

$\begin{cases} u_m = R i_m \\ \varphi = 0 \end{cases} \Rightarrow u(t)$ et $i(t)$ en phase.



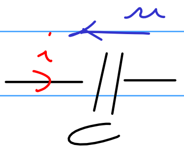


2.3. Aspect énergétique

$\underline{Z} = R$: purement résistif i.e. toute l'énergie reçue est dissipée.

3. Condensateur idéal

3.1. Impédance



$$i(t) = C\dot{u}(t) \overset{R.S.}{\Longrightarrow} \underline{i} = C\dot{\underline{u}}$$

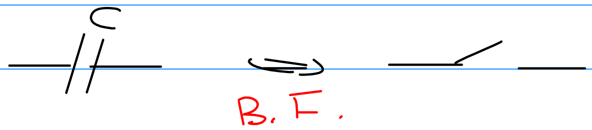
$$\underline{i} = jC\omega\, \underline{u}$$

Or par déf $\underline{u} = \underline{Z_c}\,\underline{i}$ donc $\boxed{\underline{Z_c} = \dfrac{1}{jC\omega}}$

3.2. Comportement hautes et basses fréquences

B.F. : $\omega \longrightarrow 0$: $\lim\limits_{\omega \to 0} Z_c = \lim\limits_{\omega \to 0} \dfrac{1}{C\omega} = +\infty$

Or $i_m = \dfrac{u_m}{Z_c} \Rightarrow \forall\, u_m$ (finie), $i_m = 0$



H.F : $\omega \longrightarrow +\infty$ $\lim\limits_{\omega \to +\infty} Z_c = \lim\limits_{\omega \to +\infty} \dfrac{1}{C\omega} = 0$

Or $u_m = Z_c\, i_m$ donc $\forall\, i_m$ (finie), $u_m = 0$

3.3. Lien tension / courant

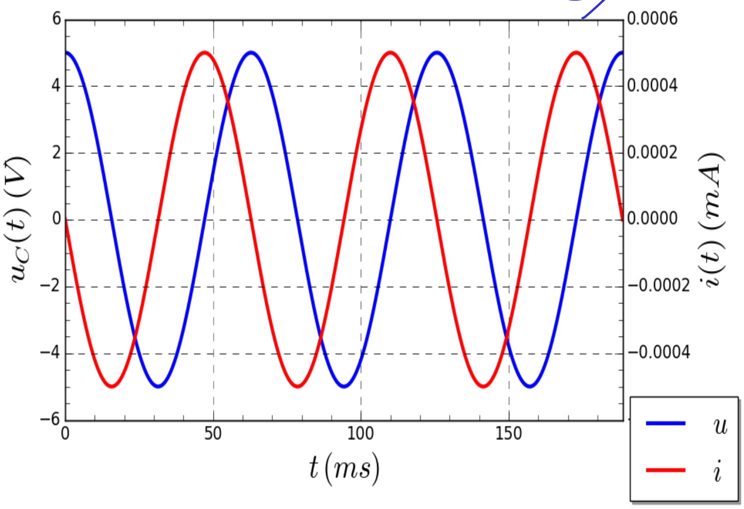
$i(t) = i_m \cos(\omega t)$

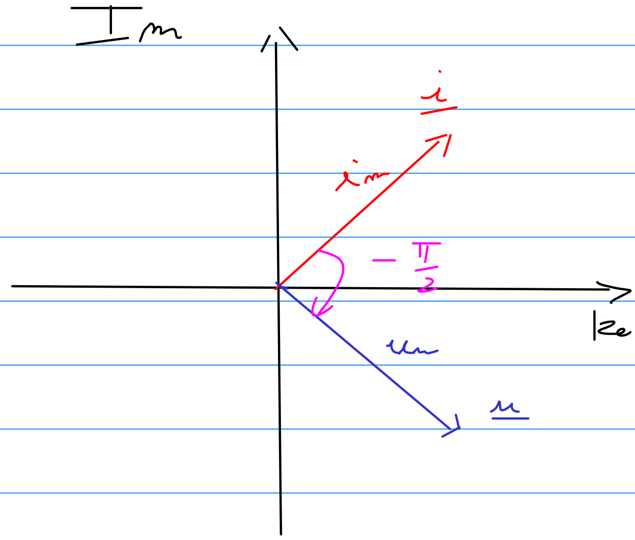
$u(t) = u_m \cos(\omega t + \varphi)$? u_m ? φ ?

$\underline{u_m} = \underline{Z_c}\,\underline{i_m}$

$$\Leftrightarrow \begin{cases} u_m = Z_c\, i_m \Leftrightarrow u_m = \dfrac{i_m}{C\omega} \\ \arg(\underline{u_m}) = \arg(\underline{Z_c}) + \underbrace{\arg(\underline{i_m})}_{0} \\ \boxed{\varphi = -\dfrac{\pi}{2}} \end{cases}$$

$u_C(t) = U_m \cos\left(\omega t - \frac{\pi}{2}\right)$





3.4. Aspect énergétique

$\underline{Z}_C = \frac{1}{jC\omega} = -j\frac{1}{C\omega}$: dipôle purement réactif et donc ne dissipe pas d'énergie.

4. Bobine idéale

4.1. Impédance

i, L, u

$\underline{u} = \underline{Z}\,\underline{i}$, $\underline{Z}$? $\quad u = L\dfrac{di}{dt} \Rightarrow \underline{u} = L\dfrac{d\underline{i}}{dt} = jL\omega\,\underline{i}$

donc $\boxed{\underline{Z} = jL\omega}$

4.2. Comportement énergétique

La bobine est un dipôle purement réactif donc elle ne dissipe pas d'énergie.

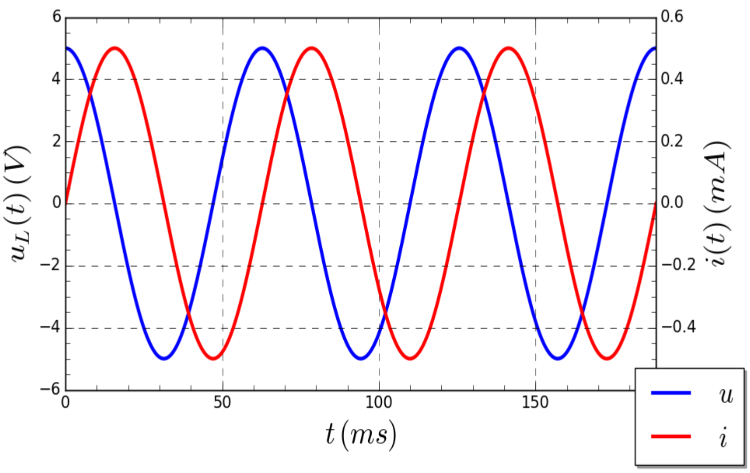
4.3. Lien entre tension et courant

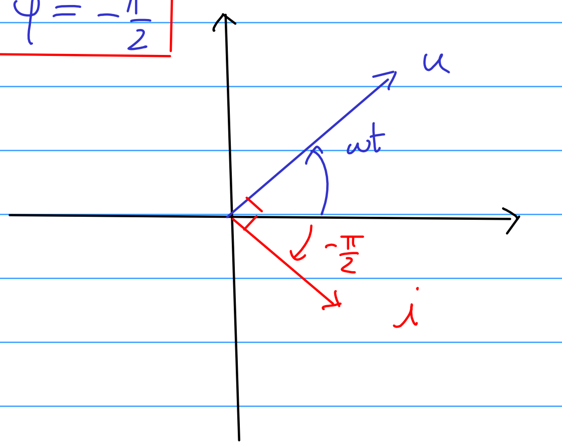
$u(t) = U_m \cos(\omega t) \Rightarrow \underline{u} = \underline{u_m}\, e^{j\omega t}$, $\underline{u_m} = u_m$

$i(t) = i_m \cos(\omega t + \varphi) \Rightarrow \underline{i} = \underline{i_m}\, e^{j\omega t}$, $\underline{i_m} = i_m e^{j\varphi}$

$\underline{u} = \underline{Z}\,\underline{i} \Rightarrow \underline{u_m} = \underline{Z}\,\underline{i_m} \Leftrightarrow \begin{cases} \boxed{U_m = Z\, i_m = L\omega\, i_m} \\ 0 = \underbrace{\arg(\underline{Z})}_{+\frac{\pi}{2}} + \varphi \end{cases}$

$\Rightarrow \boxed{\varphi = -\dfrac{\pi}{2}}$





4.4. Comportement asymptotique

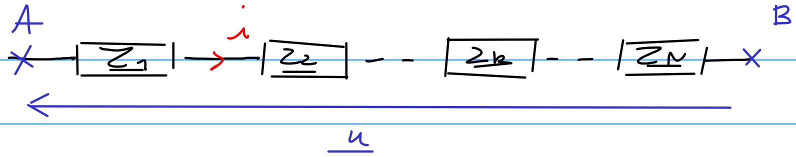
Si $\omega \to 0$ (B.F. / Régime stationnaire) : $Z \to 0 \Rightarrow \forall i,\ u = 0$ soit (bobine) $\Leftrightarrow$ (fil)

Si $\omega \to +\infty$ (H.F.) : $Z \to +\infty \Rightarrow \forall u,\ i = 0$ soit (bobine) $\Rightarrow$ (interrupteur ouvert)

(inertie de la bobine)

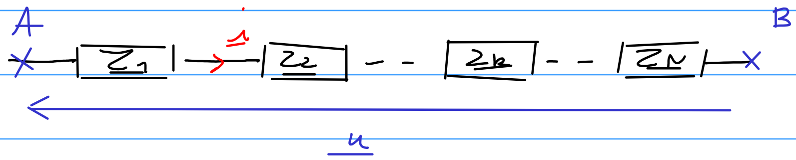
4. Associations d'impédances

4.1. Association en série

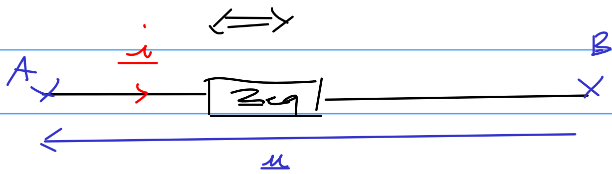


4.1.1. Impédance équivalente

(1)



(2)

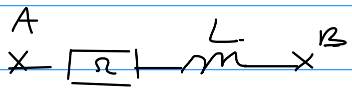


(1) $\underline{u} = \sum_{k=1}^{N} \underline{Z_k}\,\underline{i} = \left(\sum_{k=1}^{N} \underline{Z_k}\right)\underline{i}$

(2) $\underline{u} = \underline{Z_{eq}}\,\underline{i}$

$$\boxed{\underline{Z_{eq}} = \sum \underline{Z_k}}$$

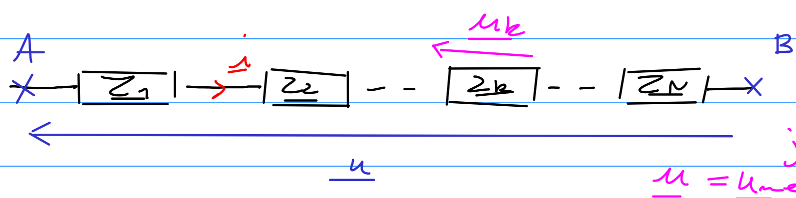
Application :



$\underline{Z_{AB}} = ?$

$$\underline{Z_{AB}} = r + jL\omega.$$

4.1.2. Pont diviseur de tension



$\underline{u} = \underline{U_m} e^{j\omega t}$

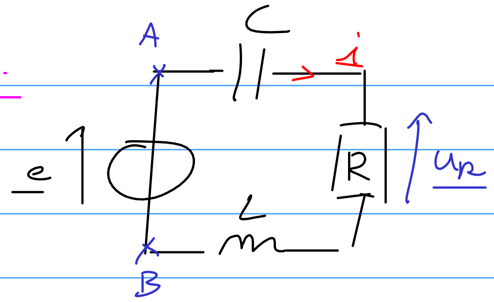
$\underline{u_k} = \underline{U_{km}} e^{j\omega t}$

$$\boxed{\underline{u_k} = \frac{\underline{Z_k}}{\sum_j \underline{Z_j}}\,\underline{u}}$$

$$\boxed{\underline{U_{km}} = \frac{\underline{Z_k}}{\sum_j \underline{Z_j}}\,\underline{U_m}}$$

Preuve : voir S9.

Application :



$$e = e_m \cos \omega t$$

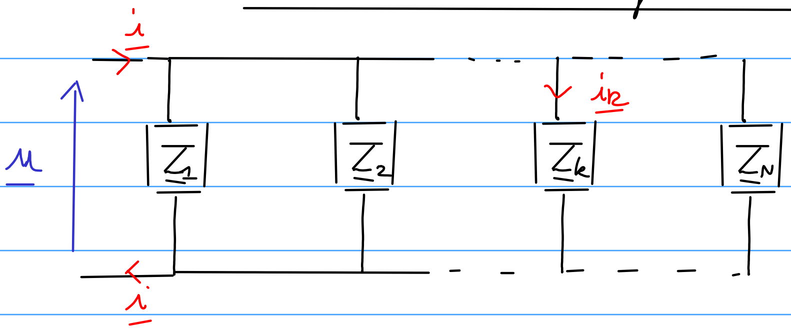
$$u_R = u_{Rm} \cos(\omega t + \varphi)$$

$$u_{Rm} = ?$$

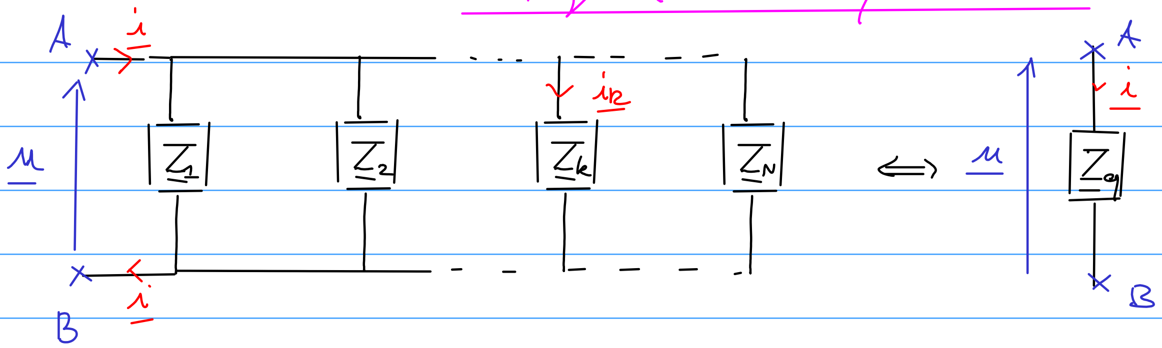
$$u_{Rm} = |\underline{u}_{Rm}| \quad \text{avec} \quad \underline{u}_{Rm} = \frac{R}{R + jL\omega + \frac{1}{jC\omega}} \underline{e}_m$$

$$\text{D'où} \quad u_{Rm} = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} e_m$$

4.2. Association parallèle



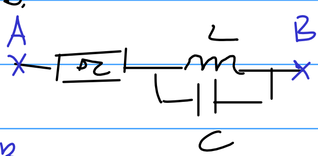
4.2.1 Impédance équivalente



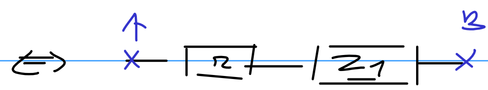
$$\boxed{\frac{1}{\underline{Z}_{eq}} = \sum_k \frac{1}{\underline{Z}_k} \quad \text{ou} \quad \underline{Y}_{eq} = \sum_k \underline{Y}_k}$$

Preuve : voir S9

Application :



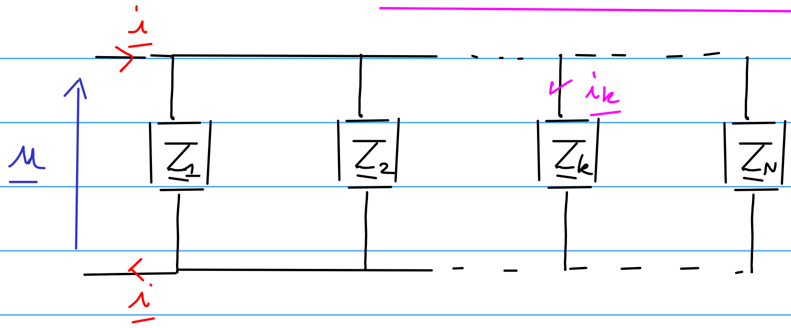
$$\underline{Z}_{AB} = ?$$



$$\text{avec} \quad \frac{1}{\underline{Z}_1} = \frac{1}{jL\omega} + jC\omega = j\left(C\omega - \frac{1}{L\omega}\right)$$

$$\text{D'où} : \quad \underline{Z}_{AB} = r - j\left(C\omega - \frac{1}{L\omega}\right)^{-1}$$

4. 2. 2. Pont diviseur de courant



$$\boxed{\underline{i}_k = \frac{\underline{Y}_k}{\sum_j \underline{Y}_j}\,\underline{i}}$$