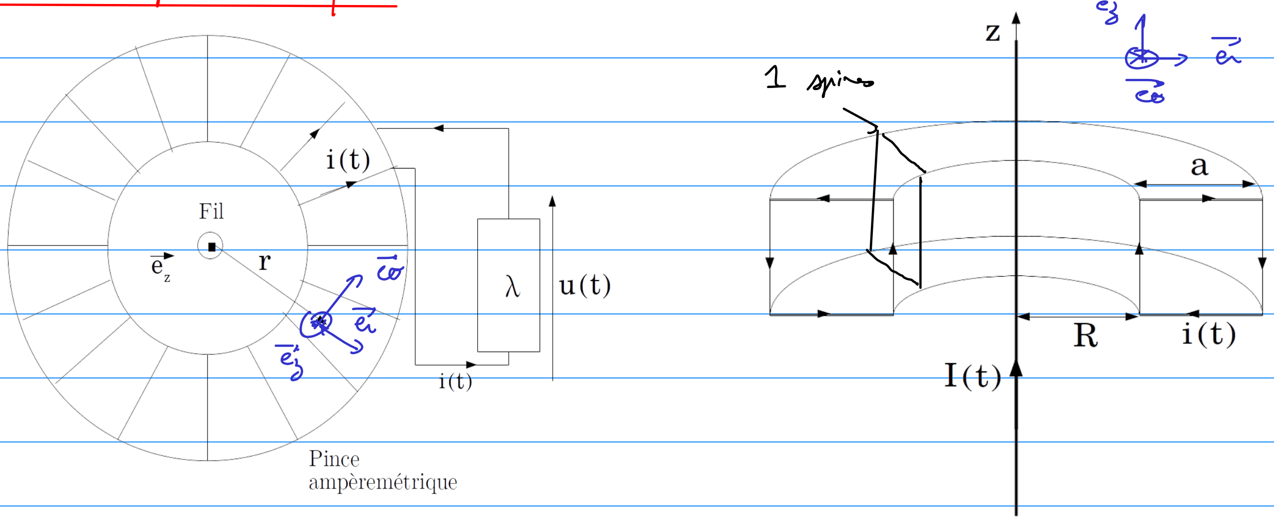
TD EM2 - Corrigé

EM1- Pince ampèremétrique



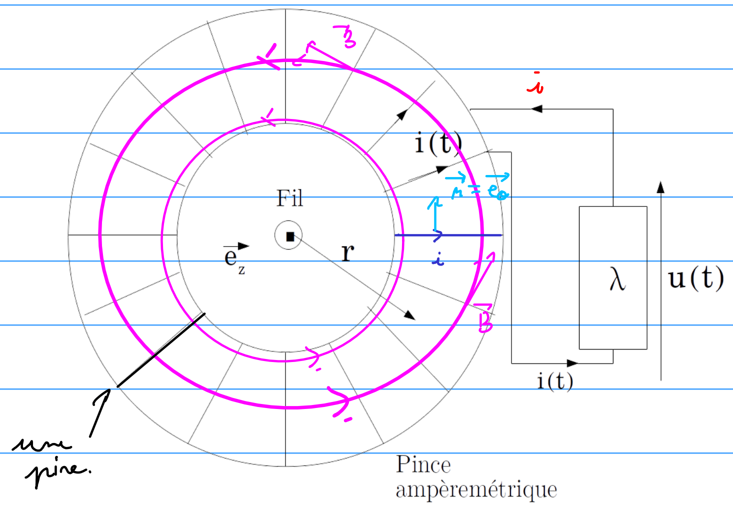
1.1/ $\vec{B} = \frac{\mu_0 I}{2\pi r}\vec{e_\theta}$. $B \nearrow$ qd $I \nearrow$

$B \searrow$ qd $r \nearrow$

1.2./

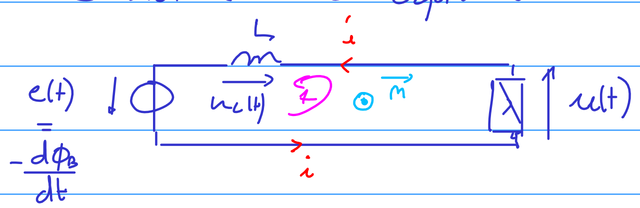
N spires.

carrées.



2.1/ Equa diff sur $u(t)$?

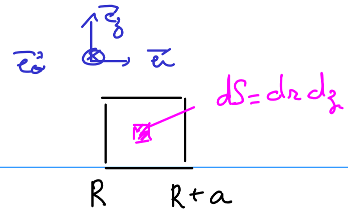
Schéma élec équivalent.



ϕ_B : flux du champ magnétique engendré par le fil

$$e(t) - u_L(t) + u(t) = 0$$

$e(t) = -\frac{d\phi_B}{dt}$, $u_L(t) = L\frac{di}{dt}$, $i(t) = -\frac{u(t)}{\lambda}$ (conv générateur)



$$\Phi_B = \iint_C \vec{B}\cdot\vec{dS} = \iint \frac{\mu_0 I}{2\pi r}\vec{e_\theta}\cdot dS\,\vec{e_\theta} = \frac{\mu_0 I}{2\pi}\int_0^a\int_R^{R+a}\frac{dr\,dz}{r}$$

$$= \frac{\mu_0 I}{2\pi}\int_0^a dz\int_R^{R+a}\frac{dr}{r} = \frac{\mu_0 I}{2\pi}a\ln\left(1+\frac{a}{R}\right) = M\times I$$

$\boxed{\Phi_B = MI}$ avec $\boxed{M = \frac{\mu_0}{2\pi}a\ln\left(1+\frac{a}{R}\right)}$

$$\Rightarrow e(t) = -M\frac{dI}{dt}$$

$$u_L(t) = L\frac{di}{dt} = -\frac{L}{\lambda}\frac{du}{dt}$$

$$e(t) - u_L(t) + u(t) = 0 \Leftrightarrow -M\frac{dI}{dt} + \frac{L}{\lambda}\frac{du}{dt} + u(t) = 0$$

$$\Leftrightarrow \frac{du}{dt} + \frac{\lambda}{L}u(t) = \frac{M\lambda}{L}\frac{dI}{dt}$$ avec $L = N.M.$

syst d'ordre 1 excitation par induction

$$\Leftrightarrow \frac{du}{dt} + \underbrace{\frac{\lambda}{NM}}_{1/\tau}u(t) = \frac{\lambda}{N}\frac{dI}{dt}. \Leftrightarrow \boxed{\dot{u} + \frac{u}{\tau} = \frac{\lambda}{N}\frac{dI}{dt}} \quad (1)$$

$$\boxed{\tau = \frac{NM}{\lambda}}$$

2.2./ $\underline{T} = \dfrac{\underline{u}}{\underline{I}}$, réponse harmonique de la pince

$$u(t) = u_m\cos(\omega t+\varphi)$$
$$I(t) = I_m\cos(\omega t)$$

$$(1) \Rightarrow \underline{\dot{u}} + \frac{\underline{u}}{\tau} = \frac{\lambda}{N}\frac{d\underline{I}}{dt}$$

$$\Leftrightarrow j\omega\underline{u} + \frac{\underline{u}}{\tau} = \frac{\lambda}{N}j\omega\underline{I} \Leftrightarrow \underline{u}\left(1/\tau + j\omega\right) = \frac{\lambda}{N}j\omega\underline{I}$$

$$\Leftrightarrow \underline{T} = \frac{\lambda/N\,j\omega}{1/\tau + j\omega}$$

$$\Leftrightarrow \boxed{\underline{T} = \frac{\lambda/N\,j\omega\tau}{1+j\omega\tau}}$$

Filtre passe-haut d'ordre 1 de fréq de coupure $1/\tau$

Mesure de $\underline{I}$? On veut $|\underline{T}| \to 1 \Leftrightarrow u_m = \frac{\lambda}{N} I_m$

i.e. $f \in$ bande passante $[f_c, +\infty[$

Il faut $j\omega\tau \to +\infty$ i.e. $f \gg f_c = \frac{1}{\tau}$.

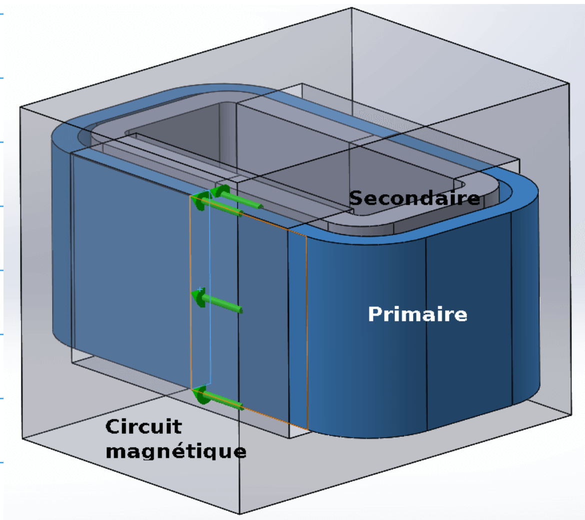
2.3. Alors $u_m = \frac{\lambda}{N} I_m$ or $u_{eff} = \frac{u_m}{\sqrt{2}}$ et $I_{eff} = \frac{I_m}{\sqrt{2}}$

$$\Rightarrow \boxed{I_{eff} = \frac{N}{\lambda} u_{eff}}$$

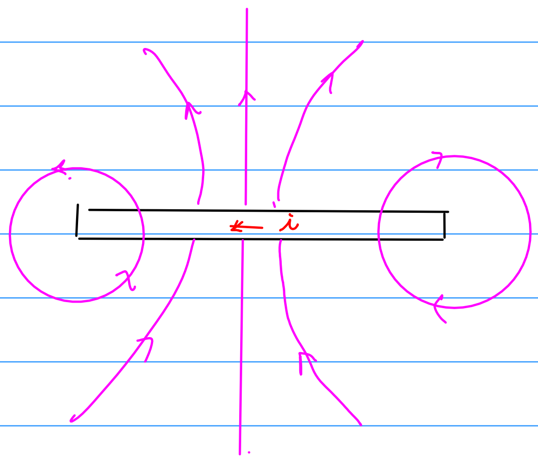
$\underline{T} = \frac{\lambda/N \; j\omega\tau}{1+j\omega\tau}$. Bande passante $f \gg f_c \Leftrightarrow \omega\tau \gg 1$

$$\underline{T} = \frac{\lambda/N \; j\omega\tau}{1+j\omega\tau} \approx \frac{\lambda/N \; j\omega\tau}{j\omega\tau} \simeq \lambda/N$$

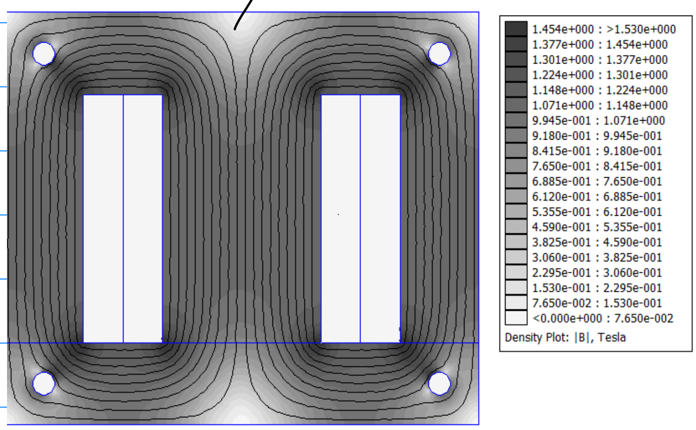
EM2 - Transformateur parfait



1)

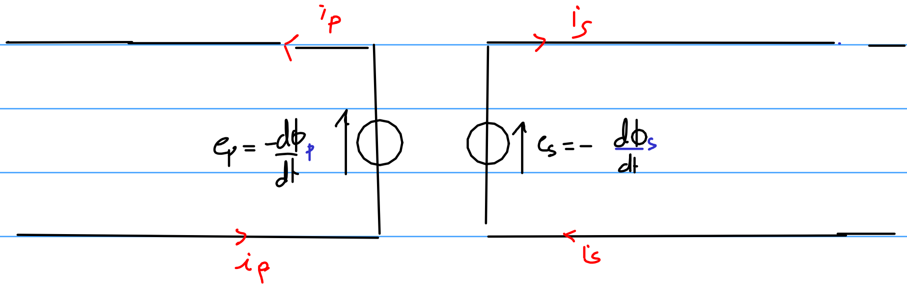


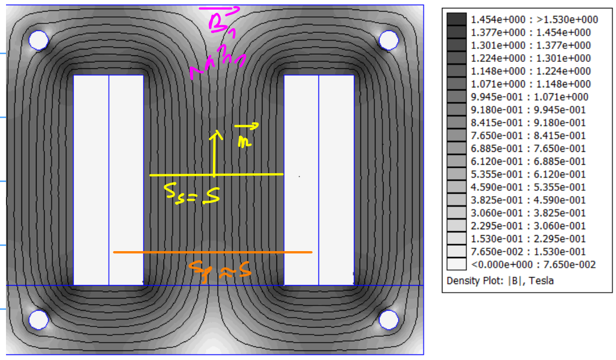
Ferromagnétique



Le circuit magnétique canalise les lignes de champ.

2) Loi des tensions





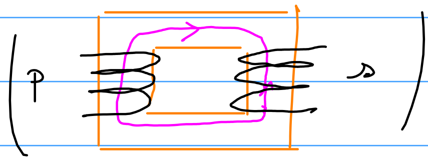
φ : flux à travers 1 spire.

$\Phi_p = N_p \varphi \Rightarrow e_p = -N_p \frac{d\varphi}{dt}$

$\Phi_s = -N_s \varphi \Rightarrow e_s = N_s \frac{d\varphi}{dt}$

D'où $\frac{d\varphi}{dt} = -\frac{e_p}{N_p} = \frac{e_s}{N_s}$

D'où : $\boxed{e_s = -\frac{N_s}{N_p} \times e_p}$



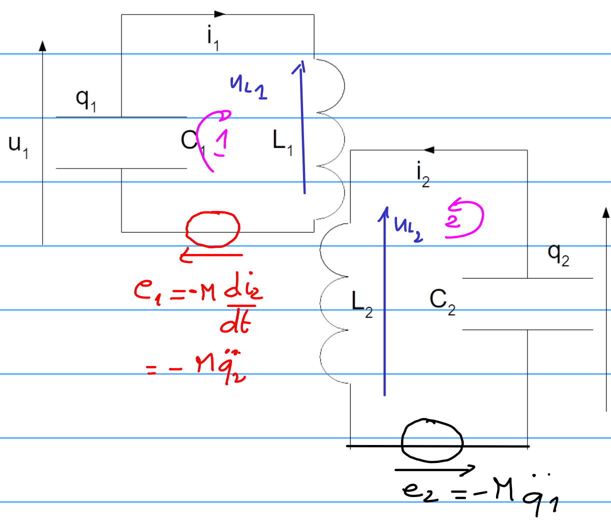
4/ $P_p = e_p i_p$

$P_s = -e_s i_s$

On $P_s = P_p$ d'où $e_p i_p = -e_s i_s = + \frac{N_s}{N_p} e_p i_s$

$\Rightarrow \boxed{i_s = \frac{N_p}{N_s} i_p}$

EM3- Oscillateurs couplés



1) Circuit 1 : $u_1 - u_{L1} + e_1(t) = 0$

$q_1 = -C_1 u_1 \Rightarrow u_1 = -\dfrac{q_1}{C_1}$

$u_{L1} = L_1 \dfrac{di_1}{dt} = L_1 \dfrac{d^2 q_1}{dt^2}$

Circuit 1 : $\ddot{q}_1 + \dfrac{1}{LC} q_1 + \dfrac{M}{L}\ddot{q}_2 = 0$ OH 1

Circuit 2 : $\ddot{q}_2 + \dfrac{1}{LC} q_2 + \dfrac{M}{L}\ddot{q}_1 = 0$ OH 2

$$\boxed{\begin{cases} \ddot{q}_1 + k\ddot{q}_2 + \omega_0^2 q_1 = 0 & (1) \\ \ddot{q}_2 + k\ddot{q}_1 + \omega_0^2 q_2 = 0 & (2) \end{cases}}$$ avec $\boxed{\omega_0 = \sqrt{\dfrac{1}{LC_1}}}$ $\boxed{k = \dfrac{M}{L}}$

Equat° couplées
≡ couplage de 2 circuits
par induction mutuelle

2/ $u = \frac{1}{2}(q_1 + q_2)$

$v = \frac{1}{2}(q_1 - q_2)$ Modes propres d'oscillation

$(1)+(2)$: $(1+k)\ddot{u} + \omega_0^2 u = 0 \Leftrightarrow \ddot{u} + \underbrace{\dfrac{\omega_0^2}{1+k}}_{\omega_1 = \frac{\omega_0}{\sqrt{1+k}} < \omega_0} u = 0$ O.H.

$(1)-(2)$: $(1-k)\ddot{v} + \omega_0^2 v = 0 \Leftrightarrow \ddot{v} + \underbrace{\dfrac{\omega_0^2}{1-k}}_{\omega_2 = \frac{\omega_0}{\sqrt{1-k}} > \omega_0} v = 0$ O.H.

Résolution :

$u(t) = A\cos(\omega_1 t) + B\sin(\omega_1 t)$ A, B ?

$v(t) = C\cos(\omega_2 t) + D\sin(\omega_2 t)$ C, D ?

Relat° de continuité :

u_1 = tension aux bornes d'un condensateur continue $\Rightarrow q_1 = C_1 u_1$ continue

u_2 = tension aux bornes d'un condensateur continue $\Rightarrow q_2 = C_2 u_2$ continue

i_1 = courant traversant la bobine continue $\Leftrightarrow \dot{q}_1$ continue

i_2 = courant traversant la bobine continue $\Leftrightarrow \dot{q}_2$ continue

$u = \frac{1}{2}(q_1 + q_2) \Rightarrow u$ continue et $\dot{u}$ continue

$v = \frac{1}{2}(q_1 - q_2) \Rightarrow v$ continue et $\dot{v}$ continue.

À $t = 0$: × $u(0^+) = u(0^-)$ avec $u(0^+) = A$

$u(0^-) = \frac{1}{2}(\underbrace{q_1(0^-)}_{Q} + \underbrace{q_2(0^-)}_{0}) = \frac{Q}{2}$

$\Rightarrow A = \frac{Q}{2}$

× $\dot{u}(0^+) = \dot{u}(0^-)$ avec $\dot{u}(0^+) = B\omega_1$

$\dot{u}(0^-) = \frac{1}{2}(\dot{q}_1(0^-) + \dot{q}_2(0^-)) = 0$

$\Rightarrow B = 0$

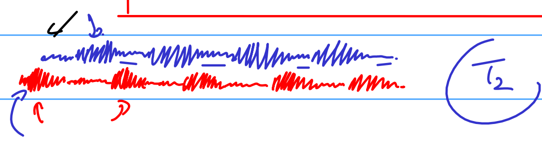
D'où : $\boxed{u(t) = \frac{Q}{2}\cos(\omega_1 t)}$

De même : $\boxed{v(t) = \frac{Q}{2}\cos(\omega_2 t)}$

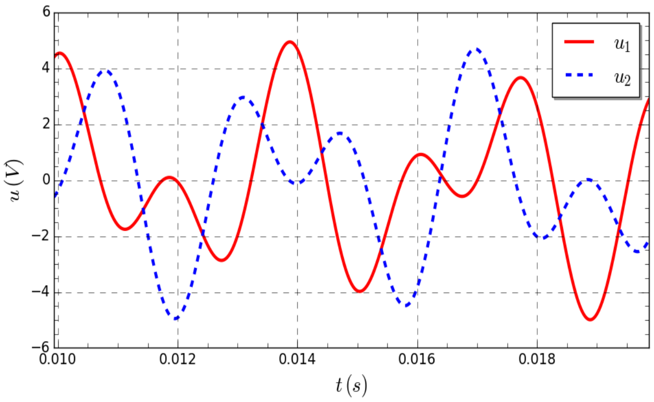
3/ $u_1(t)$, $u_2(t)$?

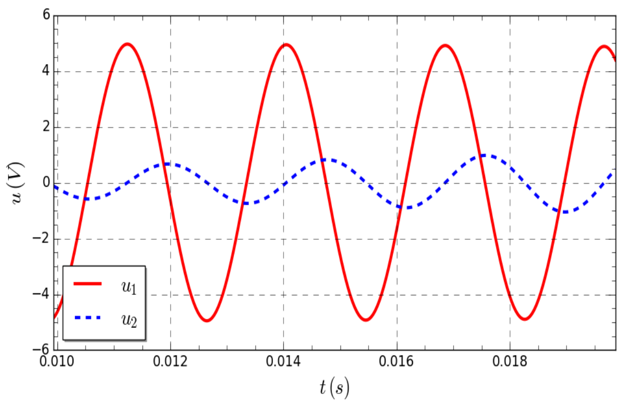
$u_1(t) = \frac{q_1}{C}$ avec $q_1 = u(t) + v(t) \Rightarrow \boxed{u_1 = \frac{Q}{2C}(\cos(\omega_1 t) + \cos(\omega_2 t))}$

$u_2(t) = \frac{q_2}{C}$ avec $q_2 = u(t) - v(t) \Rightarrow \boxed{u_2 = \frac{Q}{2C}(\cos(\omega_1 t) - \cos(\omega_2 t))}$



4/





$k = \frac{M}{L} = 0,5$: couplage fort.

Oscillations anharmonique.

$\omega_1 = \frac{\omega_0}{\sqrt{1+k}} \lesssim \omega_0$, $\omega_2 = \frac{\omega_0}{\sqrt{1-k}} \gtrsim \omega_0$

$k = \frac{M}{L} = 0,01 \ll 1$: couplage faible.

Oscillations quasi harmonique de période $T_1 = \frac{2\pi}{\omega_1} \approx \frac{2\pi}{\omega_0}$

sur $\tau \ll T_2 = \frac{2\pi}{\omega_2}$

Possible car $k \ll 1 \Rightarrow T_2 \gg T_1$

5/ Oscillations amorties par dissipation de l'énergie électrique par effet Joule.